# The Derivative

Math 130 - Essentials of Calculus

2 October 2019

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### Definition (Derivative)

*The derivative of the function f at the number a, denoted $f'(a)$, is defined by*

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

*if the limit exists.*

Likewise, we could use the alternate form of the difference quotient to compute a derivative as well

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

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**Example**

*Compute the derivatives of the following functions at the given value:*

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1. $g(t) = t^2 + 4t$, $t = 1$
2. $f(x) = \sqrt{2x}$, $x = 2$

# (Some) Applications of the Derivative

- If $s(t)$ is a function that represents the displacement (position) of an object, then the derivative is the instantaneous rate of change of its position, i.e., it's velocity. The absolute value of its velocity is called the *speed*.

# (Some) Applications of the Derivative

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- Suppose $C(q)$ is the total cost to produce $q$ units of a good or service. Then the derivative $C'(q)$ is what is called the *marginal cost*. This basically tells us how much it costs to produce the *next* unit, which could be especially useful since the cost of producing extra units is likely to change based on just how many is desired to be produced. Likewise, we can take the derivative of a revenue function $R(q)$ to get the *marginal revenue*, which would tell us approximately how much extra income is gained by selling one extra unit.

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1. *What is the meaning of $H'(58)$? What are its units? ($58^\circ F \approx 15^\circ C$)*

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*Let $H(t)$ be the daily cost, in dollars, to heat an office building when the outside temperature is $t$ degrees Fahrenheit.*

1. *What is the meaning of $H'(58)$? What are its units? ($58^\circ F \approx 15^\circ C$)*
2. *Would you expect $H'(58)$ to be positive or negative? Why?*

# Now You Try It!

### Example

*Let $P(x)$ be the profit, in dollars, a souvenir shop makes from selling $x$ coffee mugs during a week.*

1. *Interpret the statement $P(80) = -125$.*
2. *Interpret the statement $P'(80) = 1.5$.*

# The Derivative - Revisited

Instead of taking the derivative of a function $f(x)$ at a fixed number $a$, we will now take a different approach and let the value of $a$ vary. In other words, we never plug in a value for $x$. We then get the derivative as a function of $x$.

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- The domain of $f'$ consists of the values in the domain of $f(x)$ for which the limit above exists.
- The function $f(x)$ is said to be *differentiable* at $x = a$ if the derivative $f'(a)$ exists.

# Comparing the Graphs of $f$ and $f'$

### Example

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2. $f(x) = 2x^2 - 3$

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There are three ways in which a function could not be differentiable:

- A corner
- A discontinuity
- A vertical tangent